

Time Dependent Analysis of Decays $\Lambda_b \rightarrow \Lambda + D^0$ and $\Lambda_b \rightarrow \Lambda + \bar{D}^0$

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The time-dependent analysis of the decays $\Lambda_b \rightarrow \Lambda + D^0(t)$ and $\Lambda_b \rightarrow \Lambda + \bar{D}^0(t)$ is discussed. The effect of particle mixing due to time evolution of D^0 and \bar{D}^0 on the observables for these decays viz the branching ratio of decay widths and the asymmetry parameters α, β and γ are analysed. It is shown that it is possible to extract information about $(\Delta m/\Gamma) \sin \hat{\gamma}$ from the experimental data for these observables. Here $\Delta m = m_{D_1^0} - m_{D_2^0}$, Γ is the decay widths for D 's and $\hat{\gamma}$ is the weak phase.

In this paper, we discuss the time dependent analysis of the decays $\Lambda_b \rightarrow \Lambda + D^0(t)$ and $\Lambda_b \rightarrow \Lambda + \bar{D}^0(t)$. Due to D^0 and \bar{D}^0 mixing, a pure $D^0(\bar{D}^0)$ beam acquires a component of $\bar{D}^0(D^0)$ as it evolves. We analyse the effect of particle mixing on the diservables for these decays viz the ratio of decay widths and the asymmetry parameters, α, β , and γ .

These decays have been previously studied in references [1] and [2]; especially the decays $\Lambda_b \rightarrow \Lambda + D_{1,2}$ where $D_{1,2}$ are CP-eigenstates. The decays are described by four amplitudes $A_D(t), A_{\bar{D}}(t), B_D(t)$ and $B_{\bar{D}}(t)$. Denoting these amplitudes as $R_D(t)$ and $R_{\bar{D}}(t)$, where $R = A$ or B , we get the time dependent amplitudes.

$$|R_D(t)|^2 = \frac{1}{2} e^{-\Gamma t} [(1 + \cos \Delta m t) R_D^2 - 2 \sin \Delta m t \sin \hat{\gamma} R_D R_{\bar{D}} + (1 - \cos \Delta m t) R_{\bar{D}}^2] \quad (1)$$

$$\text{Re } A_D^*(t) B_D(t) = \frac{1}{2} e^{-\Gamma t} [(1 + \cos \Delta m t) A_D B_D + \sin \hat{\gamma} \sin \Delta m t (A_D B_{\bar{D}} + A_{\bar{D}} B_D) + (1 - \cos \Delta m t) A_{\bar{D}} B_{\bar{D}}] \quad (2)$$

$$\text{Im } A_D^*(t) B_D(t) = \frac{1}{2} e^{-\Gamma t} [(1 + \cos \hat{\gamma} \sin \Delta m t) (A_{\bar{D}} B_D - A_D B_{\bar{D}})] \quad (3)$$

where we have explicitly exhibited the weak phase $\hat{\gamma}$. After taking out the weak phase, these amplitudes are real, if we neglect the final state interactions. For \bar{D} , change $D \rightarrow \bar{D}$ and $\sin \Delta m t \rightarrow -\sin \Delta m t$ in Eqs. (1), (2) and (3). We now take the time average of these amplitudes:

$$\bar{R}_D^2 = \frac{\int_0^\infty R_D^2(t) dt}{\int_0^\infty e^{-\Gamma t} dt} \quad (4)$$

After taking the time average and neglecting terms of the order $(\Delta m/\Gamma)^2$, we obtain (similar results follow if instead of taking time average, we take $t \sim \frac{1}{\Gamma}$)

$$\bar{R}_D^2 = \approx [R_D^2 + (\Delta m/\Gamma) \sin \hat{\gamma} R_D R_{\bar{D}}] \quad (5)$$

$$\bar{\alpha}_D = 2 |\vec{k}| \left[A_D B_D + \frac{1}{2} (\Delta m/\Gamma) \sin \hat{\gamma} A_D B_{\bar{D}} + A_{\bar{D}} B_D \right] / \bar{F}_D^2 \quad (6)$$

$$\bar{\beta}_D = 2 |\vec{k}| \left[(\Delta m/\Gamma) \cos \hat{\gamma} \frac{1}{2} (A_{\bar{D}} B_{\bar{D}} - A_D B_{\bar{D}}) / \bar{F}_D^2 \right] \quad (7)$$

$$\bar{\gamma}_D = \frac{[(E_\Lambda + m_\Lambda) A_D^2 - (E_\Lambda - m_\Lambda) B_D^2] + (\Delta m/\Gamma) \sin \hat{\gamma} [(E_\Lambda + m_\Lambda) A_D A_{\bar{D}} - (E_\Lambda - m_\Lambda) B_D B_{\bar{D}}]}{\bar{F}_D^2} \quad (8)$$

where

$$\begin{aligned} \bar{F}_D^2 &= (E_\Lambda + m_\Lambda) \bar{A}_D^2 + (E_\Lambda - m_\Lambda) \bar{B}_D^2 \\ &= [(E_\Lambda + m_\Lambda) A_D^2 + (E_\Lambda - m_\Lambda) B_D^2] \\ &\quad + (\Delta m/\Gamma) \sin \hat{\gamma} [(E_\Lambda + m_\Lambda) A_D A_{\bar{D}} + (E_\Lambda - m_\Lambda) B_D B_{\bar{D}}] \end{aligned} \quad (9)$$

For \bar{D} , change $(\Delta m/\Gamma)$ to $-(\Delta m/\Gamma)$ and $A_D, B_D \leftrightarrow A_{\bar{D}}, B_{\bar{D}}$ in Eqs.(5), (6), (7), (8) and (9). It is convenient to put

$$A_D = \frac{a_D}{\sqrt{E_\Lambda + m_\Lambda}}, B_D = \frac{b_D}{\sqrt{E_\Lambda - m_\Lambda}} \quad (10)$$

In terms of these amplitudes, we have from Eqs. (6 – 9),

$$\bar{F}_D^2 = (a_D^2 + b_D^2) \left[1 + (\Delta m/\Gamma) \sin \hat{\gamma} \frac{a_D a_{\bar{D}} + b_D b_{\bar{D}}}{a_D^2 a_{\bar{D}}^2} \right] \quad (11)$$

$$\bar{\alpha}_D = \frac{2a_D b_D}{a_D^2 + b_D^2} \left[1 + (\Delta m/\Gamma) \sin \hat{\gamma} \left(\frac{1}{2} \left(\frac{b_{\bar{D}}}{b_D} + \frac{a_{\bar{D}}}{a_D} \right) - \frac{a_D a_{\bar{D}} + b_D b_{\bar{D}}}{a_D^2 + a_{\bar{D}}^2} \right) \right] \quad (12)$$

$$\bar{\beta}_D = (\Delta m/\Gamma) \cos \hat{\gamma} \frac{a_{\bar{D}} b_D - a_D b_{\bar{D}}}{a_D^2 + b_D^2} \quad (13)$$

$$\bar{\gamma}_D = \frac{1}{a_D^2 + b_D^2} \left[(a_D^2 - b_D^2) + (\Delta m/\Gamma) \sin \hat{\gamma} \left((a_D a_{\bar{D}} + b_D b_{\bar{D}}) - (a_D^2 - b_D^2) \frac{(a_D^2 - b_D^2)(a_D a_{\bar{D}} + b_D b_{\bar{D}})}{a_D^2 + b_D^2} \right) \right] \quad (14)$$

For \bar{D} , change $(\Delta m/\Gamma) \rightarrow -(\Delta m/\Gamma)$, $a_D, b_D \leftrightarrow a_{\bar{D}}, b_{\bar{D}}$ in Eqs. (11), (12), (13) and (14).

To proceed further, we note that in the factorization ansatz [2]

$$a_{\bar{D}} = \frac{|V_{ub}| |V_{cs}|}{|V_{cb}| |V_{us}|} a_D \simeq \sqrt{\rho^2 + \eta^2} a_D$$

$$b_{\bar{D}} = \sqrt{\rho^2 + \eta^2} \frac{b_D}{1+x}. \quad (15)$$

$$a_D = -\frac{G_F}{\sqrt{2}} |V_{cb} V_{us}| a_2 F_D (m_{\Lambda_b} - m_{\Lambda}) g_V \sqrt{E_{\Lambda} + m_{\Lambda}} \quad (16)$$

$$b_D = \frac{G_F}{\sqrt{2}} |V_{cb} V_{us}| a_2 F_D (m_{\Lambda_b} + m_{\Lambda}) g_A \sqrt{E_{\Lambda} - m_{\Lambda}} (1+x) \quad (17)$$

Here $x = \frac{b_p}{b_f}$ and b_p is the baryon poles contribution which contributes only to b_D . $b_f = a_2 F_D (m_{\Lambda_b} + m_{\Lambda}) g_A$. Note that in Eq. (15), we have used Wolfenstein parametrization [3] of CKM matrix [4]. We will take $g_V = g_A$. Thus we can write

$$b_{\bar{D}}/a_{\bar{D}} = -d, \quad b_D/a_D = -d(1+x) \quad (18)$$

where

$$d = \frac{m_{\Lambda_b} + m_{\Lambda}}{m_{\Lambda_b} - m_{\Lambda}} \sqrt{\frac{m_{\Lambda_b} - m_{\Lambda}}{m_{\Lambda_b} + m_{\Lambda}}} \simeq 0.946 \quad (19)$$

on using $m_{\Lambda_b} = 5.624 \text{ GeV}$ and $m_{\Lambda} = 1.116 \text{ GeV}$.

Using Eqs. (18) and (15), we obtain from Eqs. (11), (12), (13) and (14).

$$\delta \equiv \frac{\Gamma(\Lambda_b \rightarrow \Lambda + \bar{D}^0)}{(\rho^2 + \eta^2) \Gamma(\Lambda_b \rightarrow \Lambda + D^0)} = \frac{1}{(\rho^2 + \eta^2)} \frac{\bar{F}_D^2}{F_D^2}$$

$$= \frac{1+d^2}{1+d^2(1+x)^2} \left[1 - (\Delta m/\Gamma) \sin \hat{\gamma} \frac{1}{\sqrt{\rho^2 + \eta^2}} \left(1 + \frac{d^2}{1+d^2} x \right) \right] \quad (20)$$

$$\bar{\alpha}_D = \frac{-2d(1+x)}{1+d^2(1+x)^2} \left[1 - \sqrt{\rho^2 + \eta^2} (\Delta m/\Gamma) \sin \hat{\gamma} \left(\frac{x}{2(1+x)} \right) \frac{1-d^2(1+x)^2}{1+d^2(1+x)^2} \right] \quad (21)$$

$$\bar{\beta}_D = -\sqrt{\rho^2 + \eta^2} (\Delta m/\Gamma) \cos \hat{\gamma} \frac{dx}{1+d^2(1+x)^2} \quad (22)$$

$$\bar{\gamma}_D = \frac{1}{1+d^2(1+x)^2} \left[1 - d^2(1+x)^2 + \sqrt{\rho^2 + \eta^2} (\Delta m/\Gamma) \sin \hat{\gamma} \frac{2d^2 x(1+x)}{1+d^2(1+x)^2} \right] \quad (23)$$

$$\bar{\alpha}_{\bar{D}} = \frac{-2d}{1+d^2} \left[1 - (\Delta m/\Gamma) \sin \hat{\gamma} \frac{1}{\rho^2 + \eta^2} \frac{x(1-d^2)}{2(1+d^2)} \right] \quad (24)$$

$$\bar{\beta}_{\bar{D}} = -\frac{(\Delta m/\Gamma) \cos \hat{\gamma}}{\sqrt{\rho^2 + \eta^2}} \frac{dx}{1+d^2} \quad (25)$$

$$\bar{\gamma}_{\bar{D}} = \frac{1-d^2}{1+d^2} + \left[1 + \frac{1}{\sqrt{\rho^2 + \eta^2}} (\Delta m/\Gamma) \sin \hat{\gamma} \frac{2d^2 x}{(1-d^4)} \right] \quad (26)$$

We first note that the interference effect is more pronounced in the observables for \bar{D} , since the admixture of D tends to enhance it by a factor $\frac{1}{\sqrt{\rho^2 + \eta^2}}$. But since $d^2 = 0.895$, the interference effect in $\bar{\alpha}_{\bar{D}}$ is negligible. Also we note that this effect vanishes in α , β and γ for $x = 0$. But there is no reason to believe that $x = 0$ as shown in reference [2]. Just to give an estimate of the effect of $D^0 \rightarrow \bar{D}^0$ mixing, using $x = -0.64$ [2], $(\rho, \eta) = (0.05, 0.36)$ [5], we get

$$\begin{aligned} 1.38 &\leq \delta \leq 2.03 \\ 0.006 &\leq \bar{\gamma}_{\bar{D}} \leq 0.104 \end{aligned} \quad (27)$$

Without interference effect $\delta = 1.70$ and $\bar{\gamma}_{\bar{D}} = 0.055$. In deriving the inequality (27), we have used the experimental [6] upper limit $|\Delta m/\Gamma| < 0.10$.

First we note that asymmetry parameter β which characterizes CP-violation is a consequence of particle mixing. The experimental upper limit on $|\Delta m/\Gamma|$ gives

$$-0.1 \leq (\Delta m/\Gamma) \sin \bar{\gamma} \leq 0.1$$

Thus if we plot δ and $\bar{\gamma}_{\bar{D}}$ as function of $(\Delta m/\Gamma) \sin \bar{\gamma}$ in the range -0.1 to 0.1 , treating x as a free parameter lying in the range $-1 < x < 1$, we may be able to extract some information for $(\Delta m/\Gamma) \sin \bar{\gamma}$ from the experimental values of δ and $\bar{\gamma}_{\bar{D}}$. Experimentally, measurement of the ratio δ should not be very difficult. In Figs. 1 and 2 we have plotted δ and $\bar{\gamma}_{\bar{D}}$ vs $(\Delta m/\Gamma) \sin \bar{\gamma}$ for the four values of x viz $x = -0.8, -0.6, -0.4, 0.4, 0.6$ and 0.8 taking $\sqrt{\delta^2 + \eta^2} = 0.36$. If $\sin \hat{\gamma}$ is too small, then $(\Delta m/\Gamma) \cos \hat{\gamma}$ may be extracted from similar plot for $\bar{\beta}_{\bar{D}}$ as shown in Fig. 3.

To conclude the mixing of $D^0 - \bar{D}^0$ has some observable effects in the decays $\Lambda_b \rightarrow \Lambda + D^0$ and $\Lambda_b \rightarrow \Lambda + \bar{D}^0$. The branching ratio δ for these decays can give some information to extract the value of $(\Delta m/\Gamma) \sin \hat{\gamma}$, provided $\sin \hat{\gamma}$ is not too small.

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Figure Captions

- 1. Plot of δ vs $(\Delta m/\Gamma) \sin \hat{\gamma}$ (cf. Eq. 20) for $x = -0.8, -0.6, -0.4, 0.4, 0.6$ and 0.8 .
- 2. Plot of $\bar{\gamma}_{\bar{D}}$ vs $(\Delta m/\Gamma) \sin \hat{\gamma}$ (cf. Eq. 26) for $x = -0.8, -0.6, -0.4, 0.4, 0.6$ and 0.8 .
- 3. Plot of $\bar{\beta}_{\bar{D}}$ vs $(\Delta m/\Gamma) \cos \hat{\gamma}$ (cf. Eq. 25) for $x = -0.8, -0.6, -0.4, 0.4, 0.6$ and 0.8 .

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